

Linear regression and Logistic regression

정다래

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Machine Learning

Supervised Learning
(Precategorized Data)

Classification

분류

Regression

회귀

Unsupervised Learning
(Unlabelled Data)

Clustering

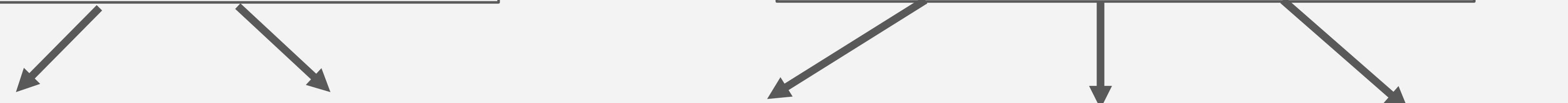
군집화

Association

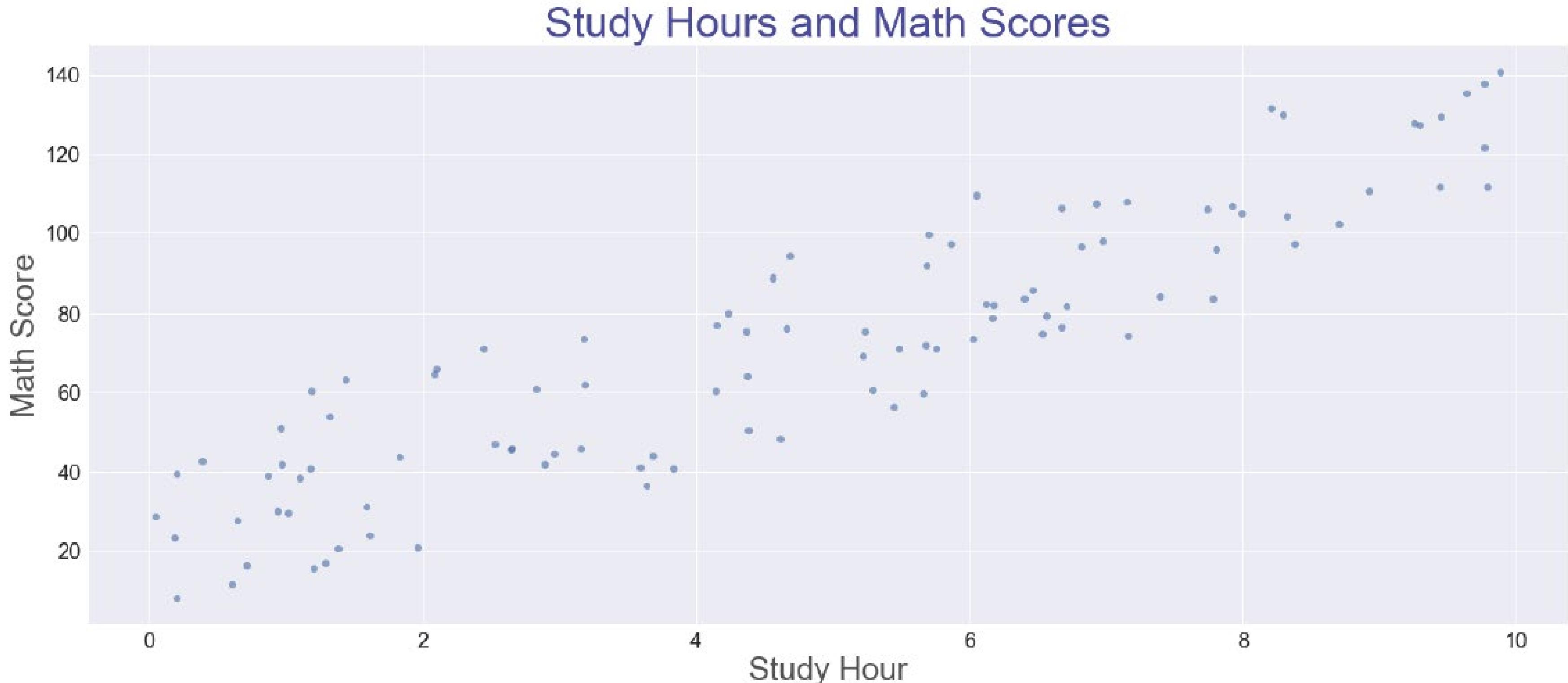
연
관

Dimension Reduction

차원축소

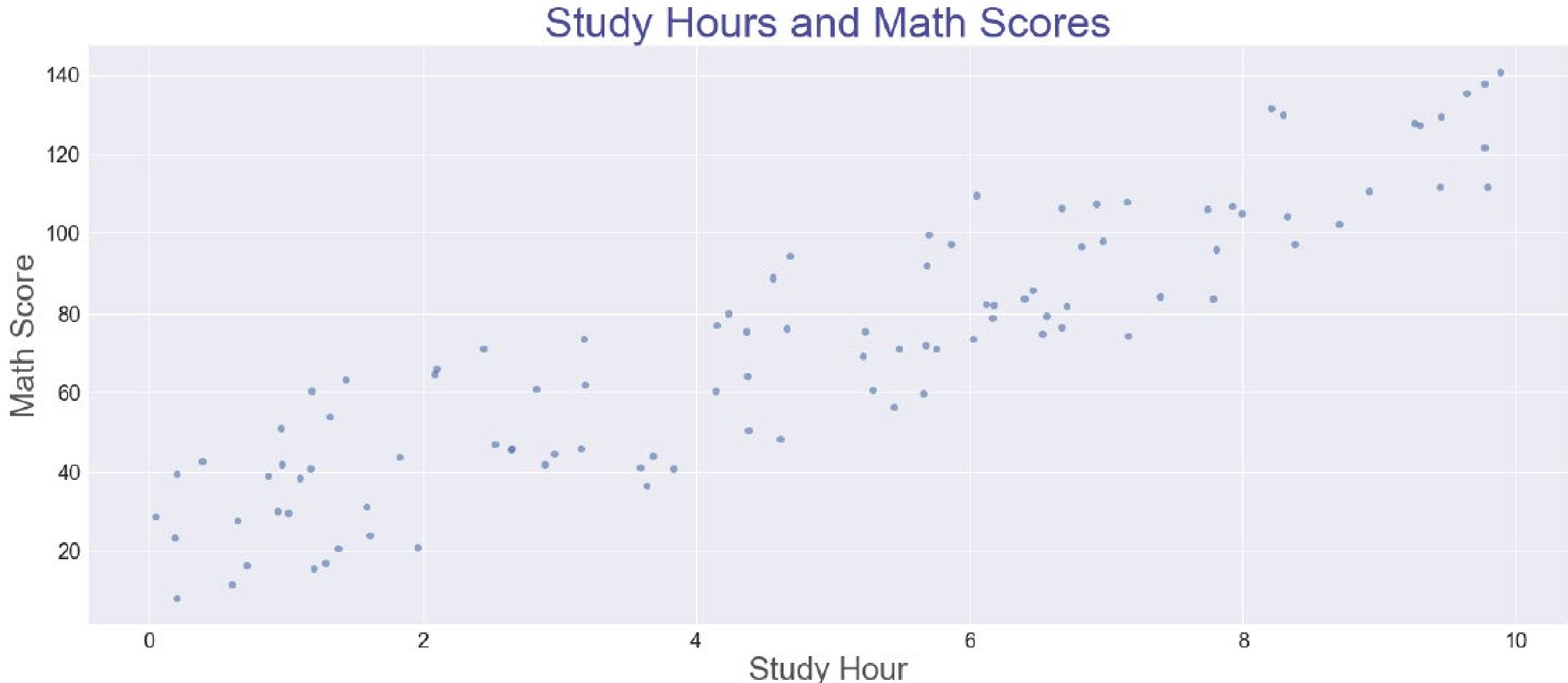


What's Regression?



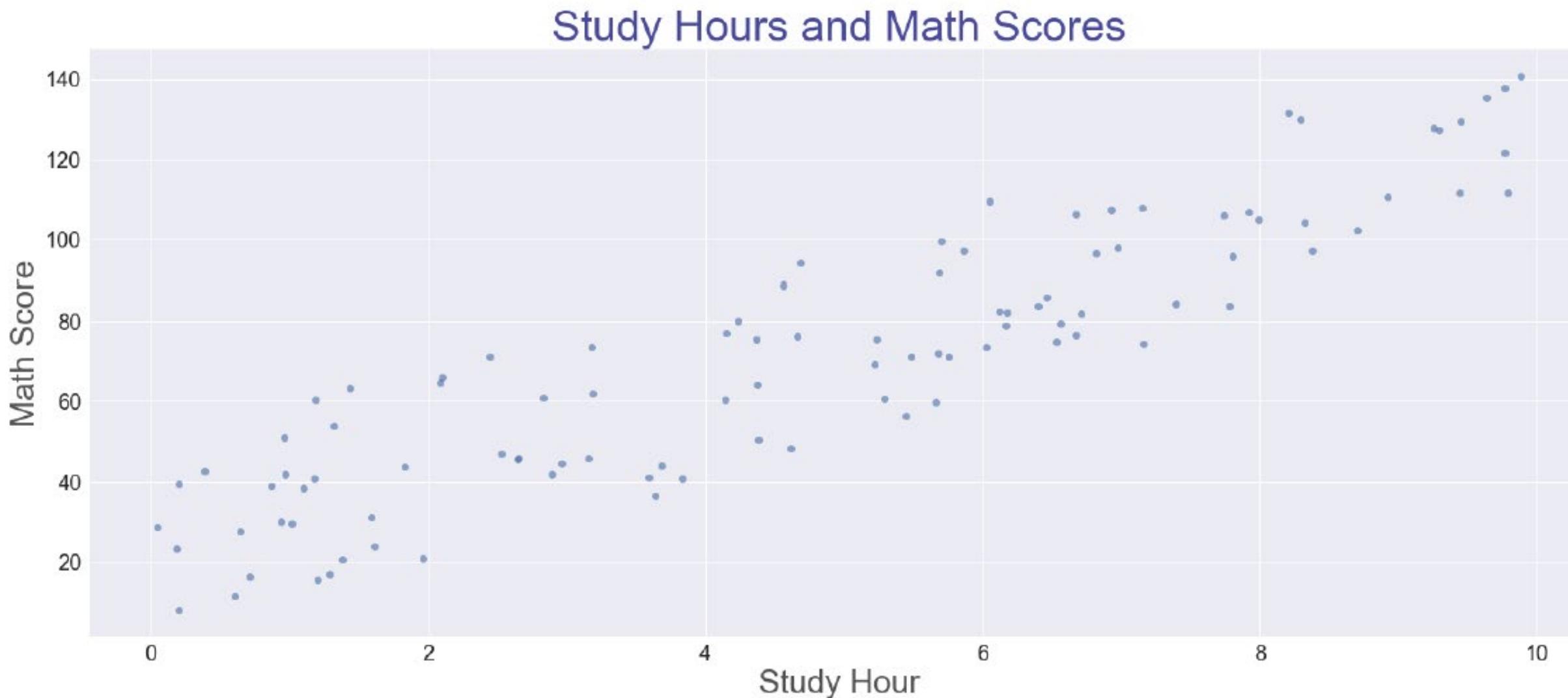
What's Regression?

predict the continuous response for **unseen** data



What's Linear Regression?

Dataset



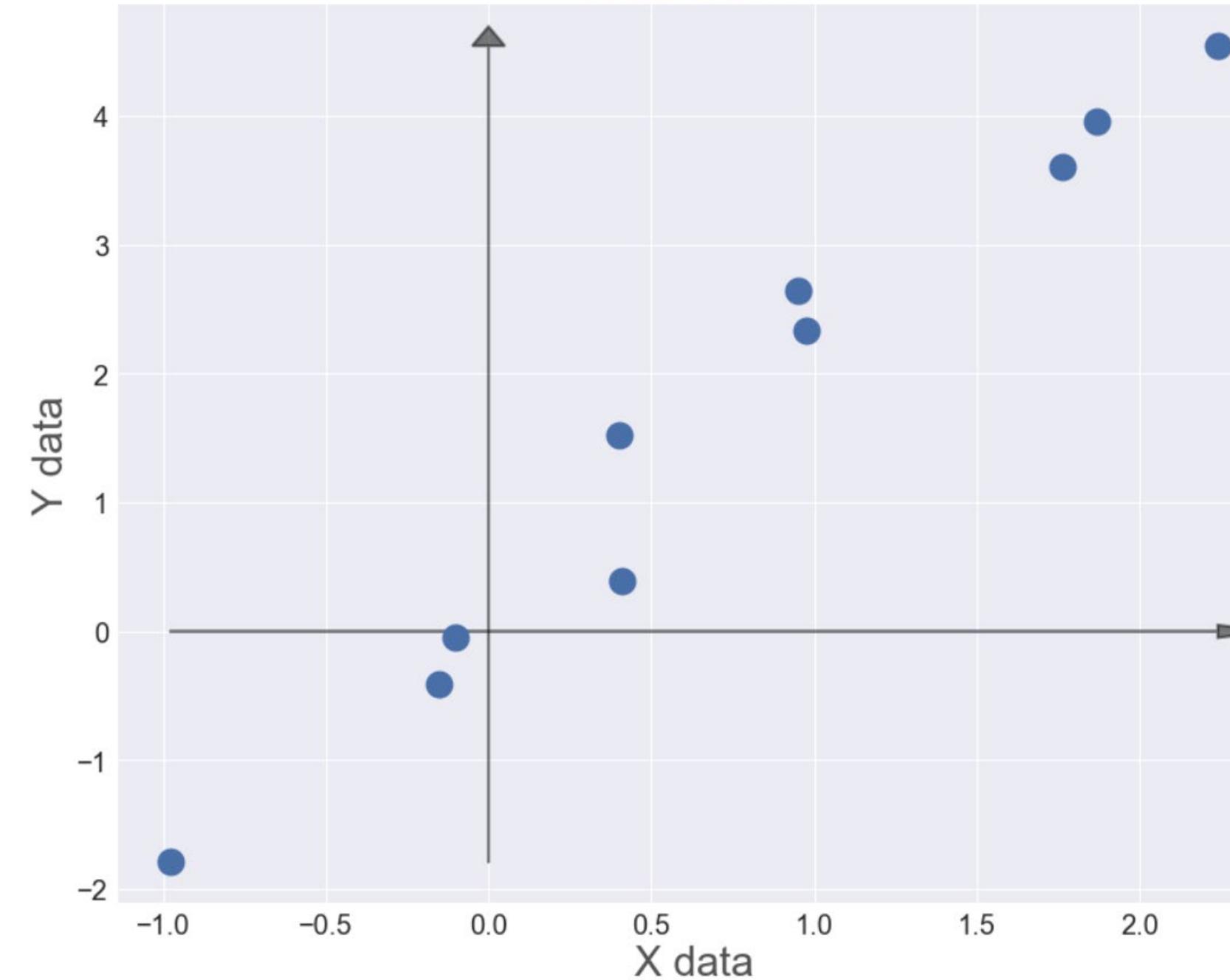
Model Setting

Learning

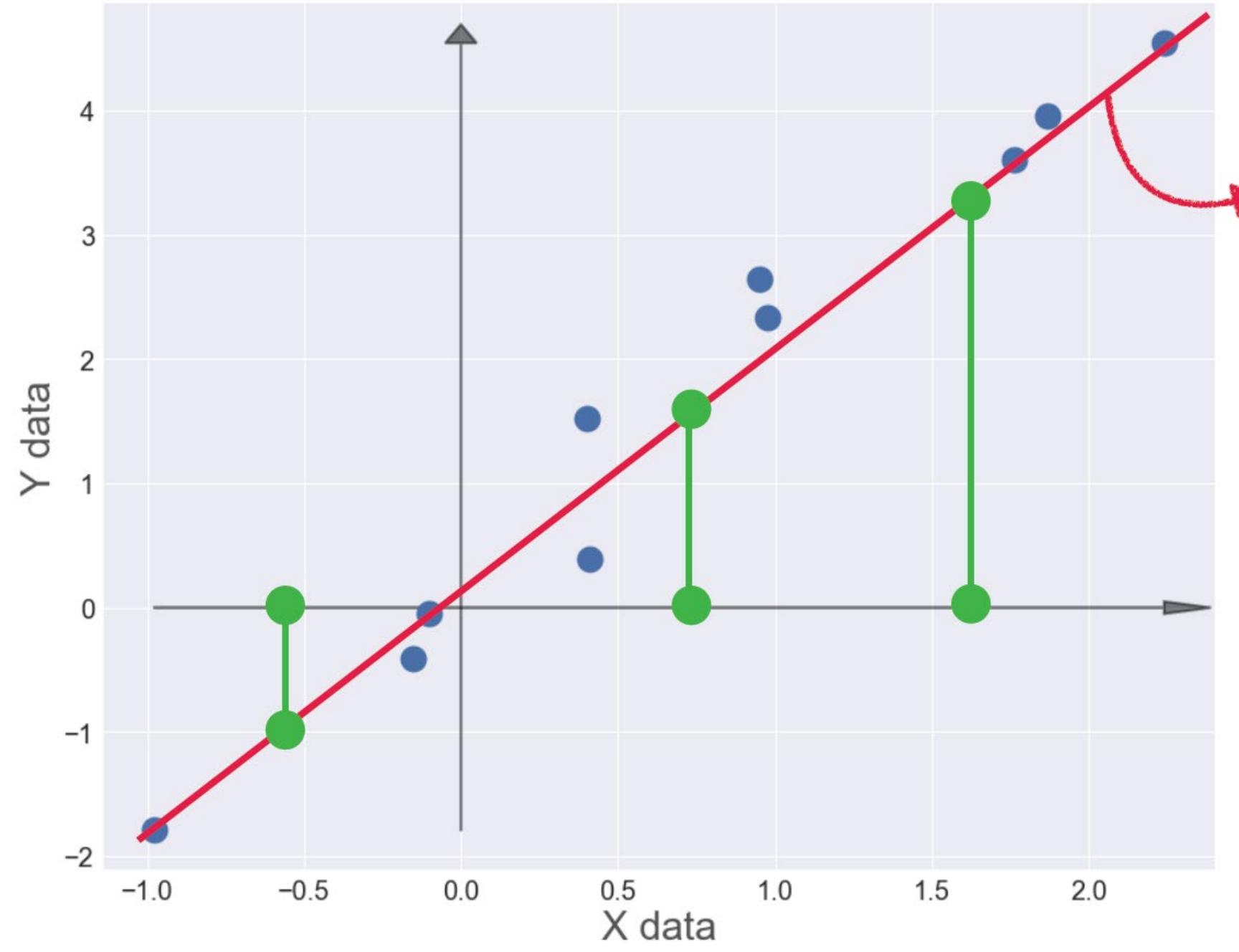
Test

Linear Regression

Dataset

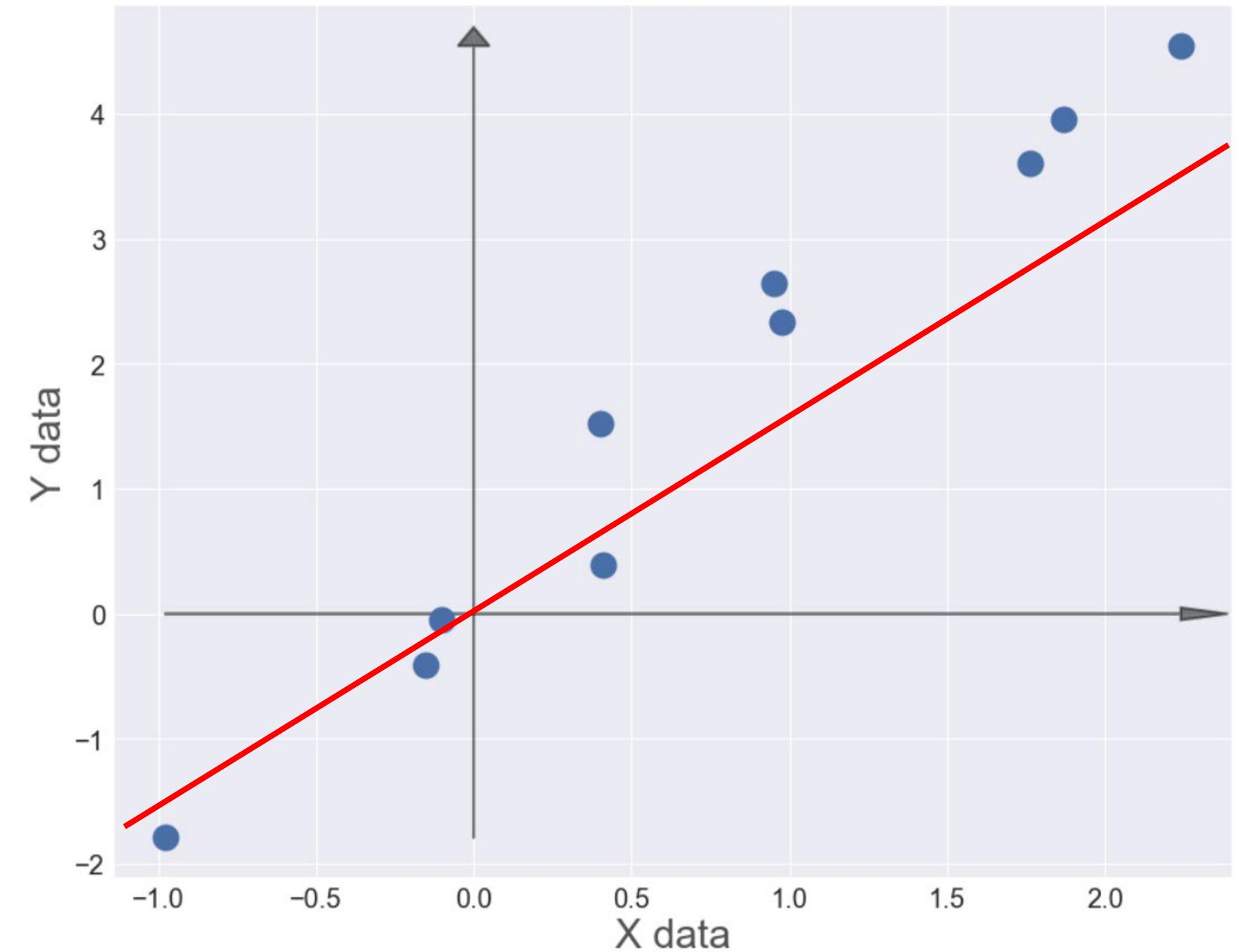


Dataset



Loss and Cost

Dataset



Loss function for one sample

(1) Data sample $(x, y) = (1, 3)$ 하나에 대한 loss function을 기술해보자.

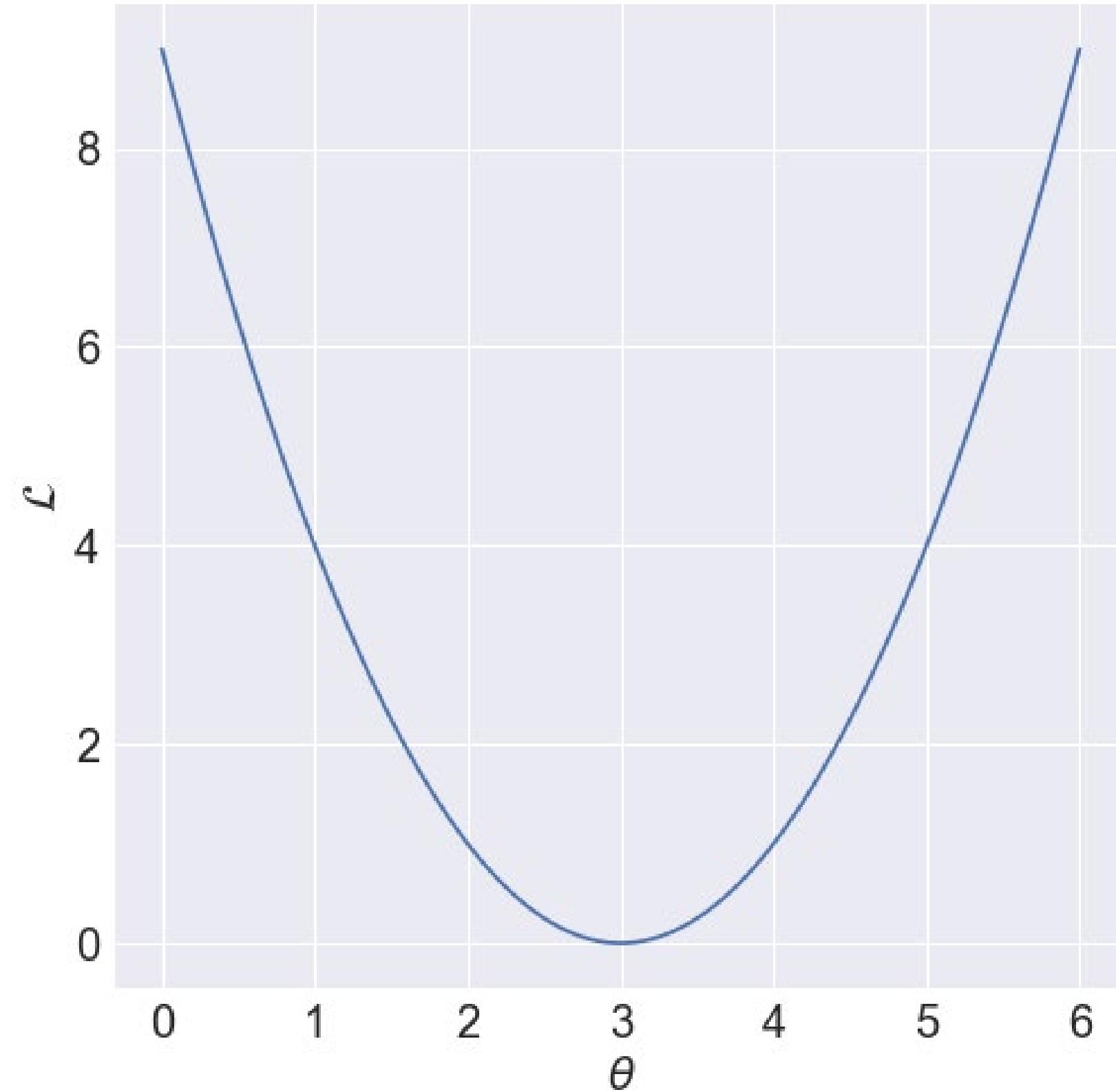
(2) Data sample $(x, y) = (1, 3)$ 하나에 대한 loss function을 그려보자.

```
import numpy as np  
import matplotlib.pyplot as plt  
  
x1 = 1  
y1 = 3*x1
```

```
th_range = np.linspace(y1 - 3, y1 + 3, 100)  
loss_funct = np.power(y1 - th_range*x1, 2)
```

```
fig, ax = plt.subplots(figsize = (5,5))  
ax.plot(th_range, loss_funct)  
ax.tick_params(axis = 'both', labelsize = 20)  
ax.set_xlabel(r'$\theta$', fontsize = 20)  
ax.set_ylabel(r'$\mathcal{L}$', fontsize = 20)
```

Loss function for one sample

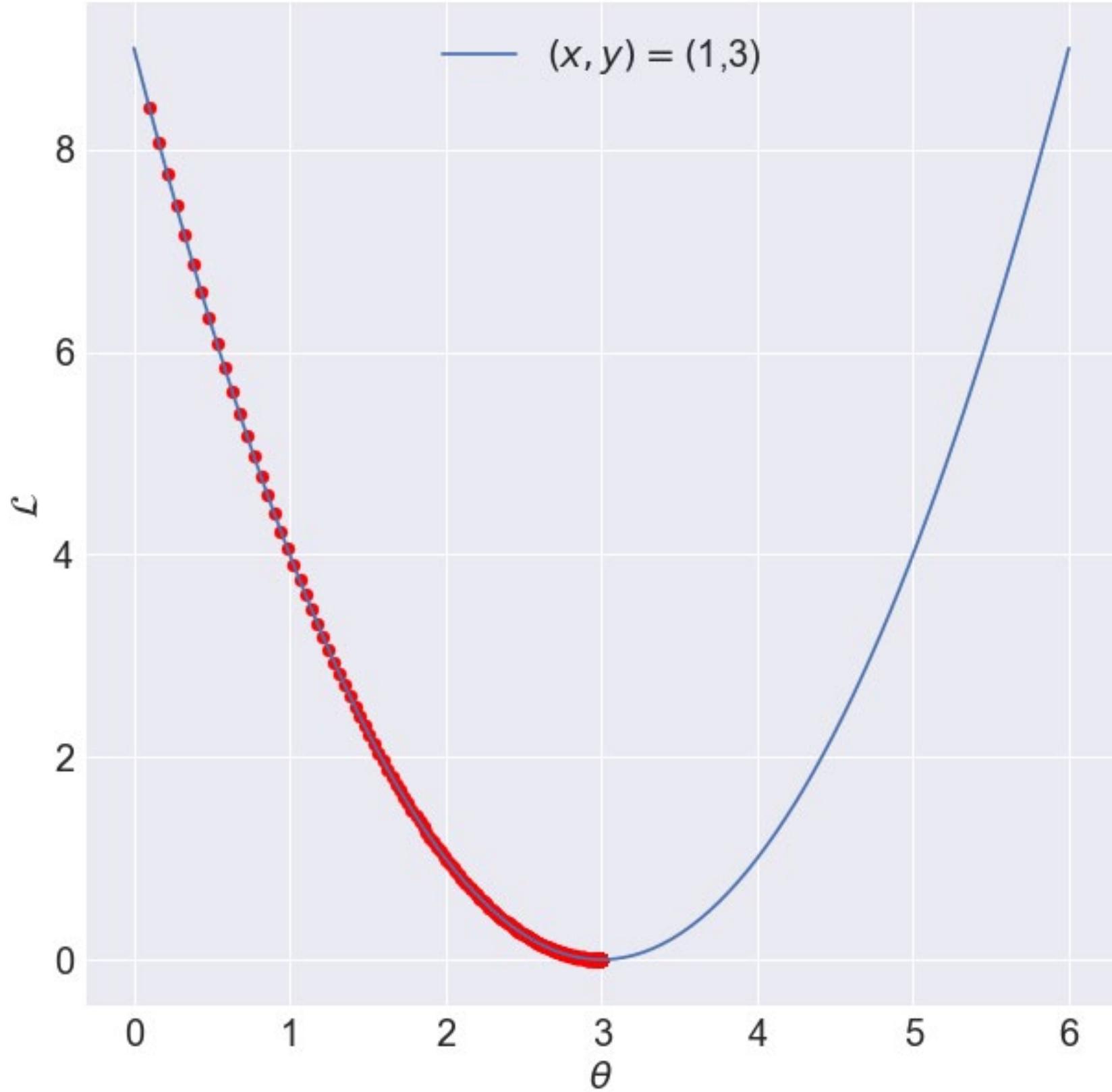


loss function에서 loss가 최소가 되는 θ 는?

다시말하면, data sample을 가장 잘 표현하는 θ 는 loss function을 최소화시키는 이다.

Learning with one sample

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Learning with one sample

위의 x_1, y_1 를 이용하여 predictor를 학습시켜보자.
data sample이 하나이므로 epoch보단 iteration이라는 표현이 적절하다.

```
th = 0.1
lr = 0.01
iterations = 500

th_list = []
loss_list = []

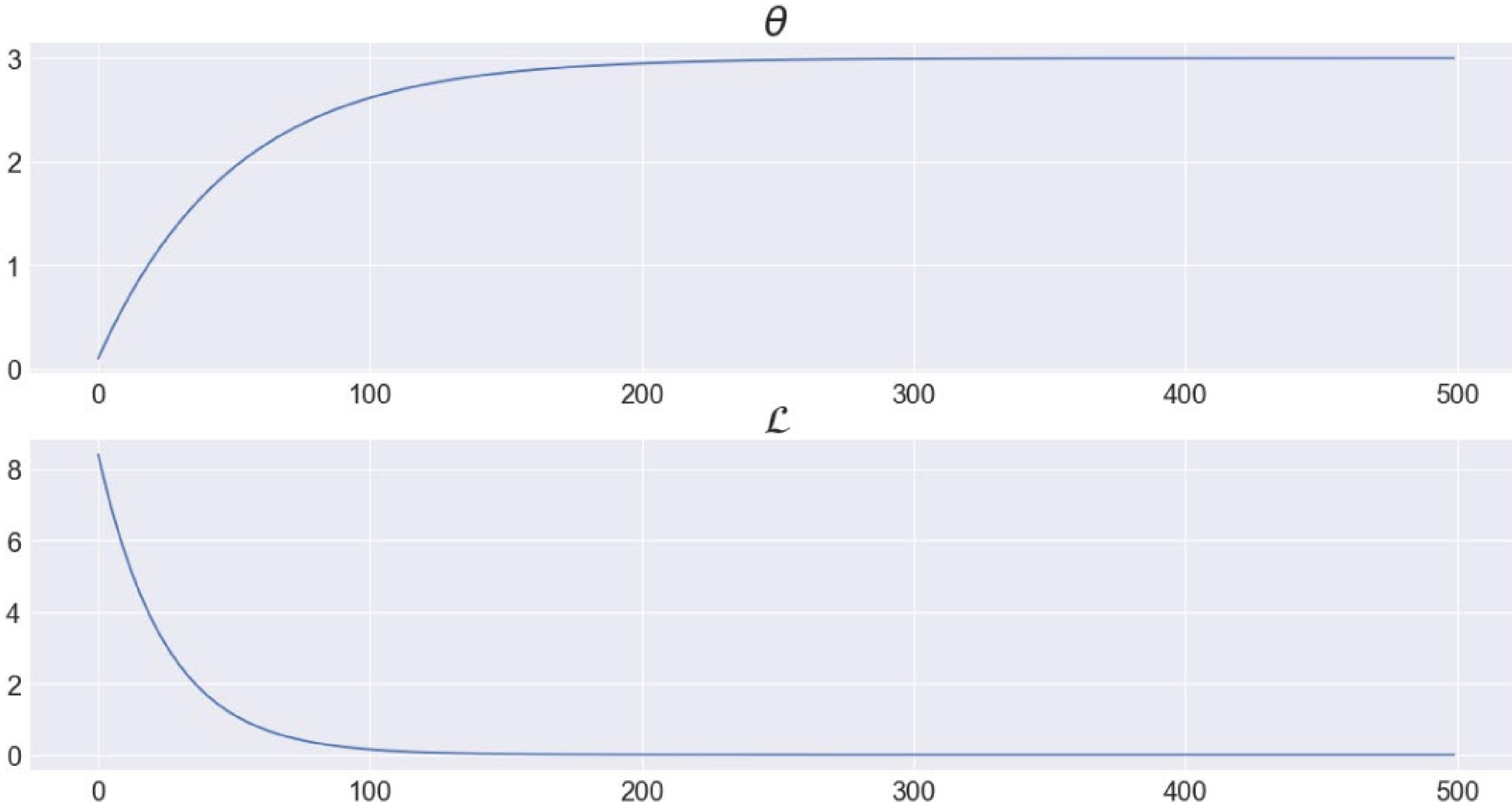
for iteration in range(iterations):
    pred = th*x1
    loss = np.power(y1 - pred, 2)

    th_list.append(th)
    loss_list.append(loss)

    th = th + 2*x1*lr*(y1 - pred)
```

```
fig, ax = plt.subplots(2, 1, figsize = (20,10))
ax[0].plot(th_list)
ax[1].plot(loss_list)
ax[0].set_title(r'$\theta$', fontsize = 30)
ax[1].set_title(r'$\mathcal{L}$', fontsize = 30)
for ax_idx in range(2):
    ax[ax_idx].tick_params(axis = 'both', labelsize = 20)
```

Learning with one sample



Learning with different data samples

Data sample $(x_1, y_1) = (0.5, 1.5)$, $(x_2, y_2) = (1, 3)$, $(x_3, y_3) = (2, 6)$ 을 이용하여 학습이 진행되는 모습을 비교해보자.

```
def get_loss_func(x, y, ax):
    th_range = np.linspace(3 - 3, 3 + 3, 100)
    loss_func = np.power(y - th_range*x, 2)

    ax.plot(th_range,
            loss_func,
            label = r'$(x,y) = ($' + str(x) + ',' + str(y)+ ')$')
    ax.tick_params(axis = 'both', labelsize = 20)
    ax.set_xlabel(r'$\theta$', fontsize = 20)
    ax.set_ylabel(r'$\mathcal{L}$', fontsize = 20)
    ax.legend(loc = 'upper center', fontsize = 20)
```

Learning with different data samples

```
def trainer(iterations, lr, x, y):
    th = 0.1
    th_list = []
    loss_list = []

    for iteration in range(iterations):
        pred = th*x
        loss = np.power(y - pred, 2)
        th_list.append(th)
        loss_list.append(loss)

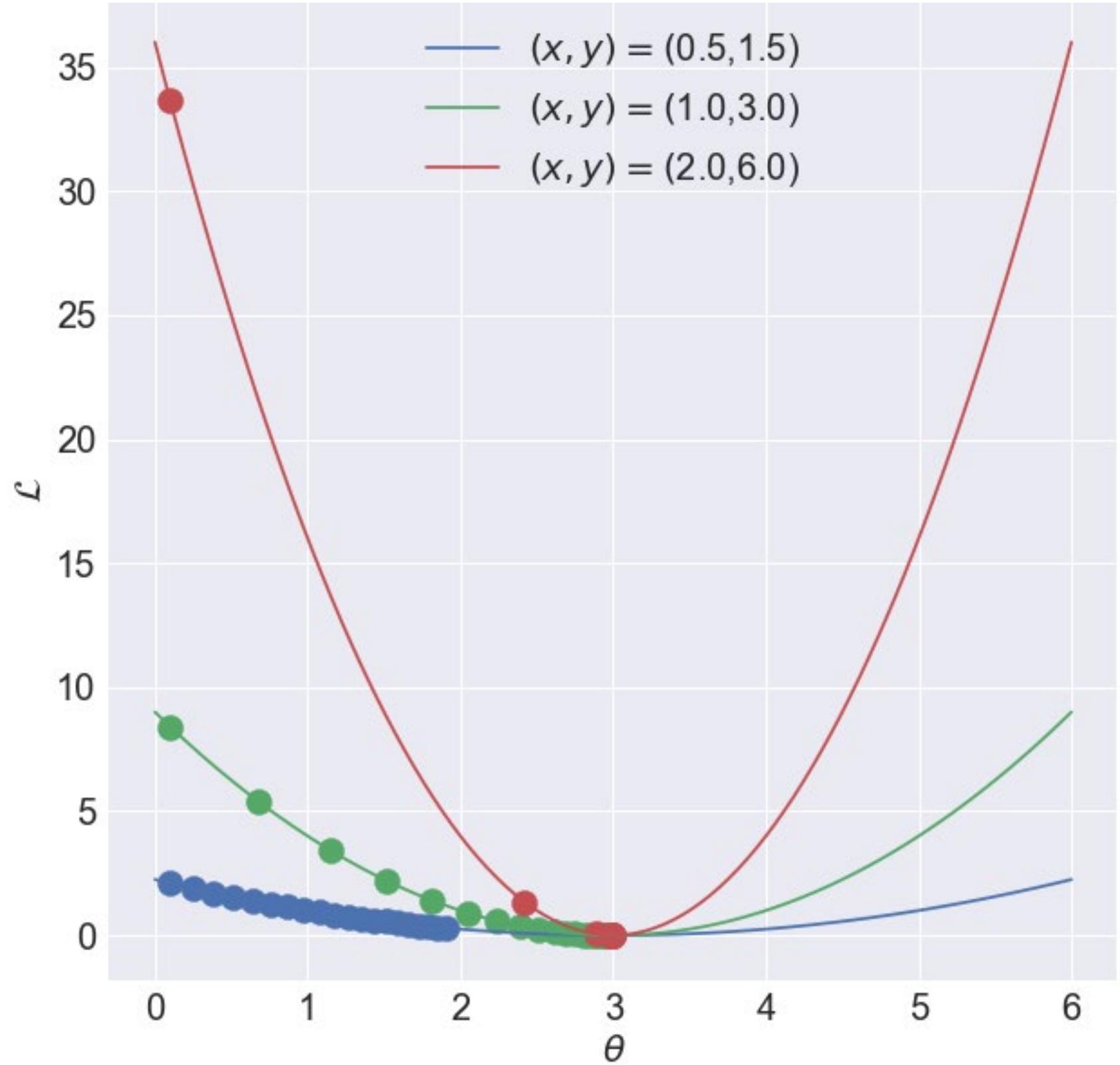
        th = th + 2*x*lr*(y - pred)
    return th_list, loss_list
```

Learning with different data samples

```
x_data = np.array([0.5, 1, 2])
y_data = 3*x_data

fig, ax = plt.subplots(figsize = (10,10))

for x, y in zip(x_data, y_data):
    get_loss_funct(x, y, ax)
    th_list, loss_list = trainer(20, 0.1, x, y)
    ax.scatter(th_list, loss_list, s = 200)
```



Learning with different data samples

```
x_data = np.array([0.5, 1, 2])
y_data = 3*x_data

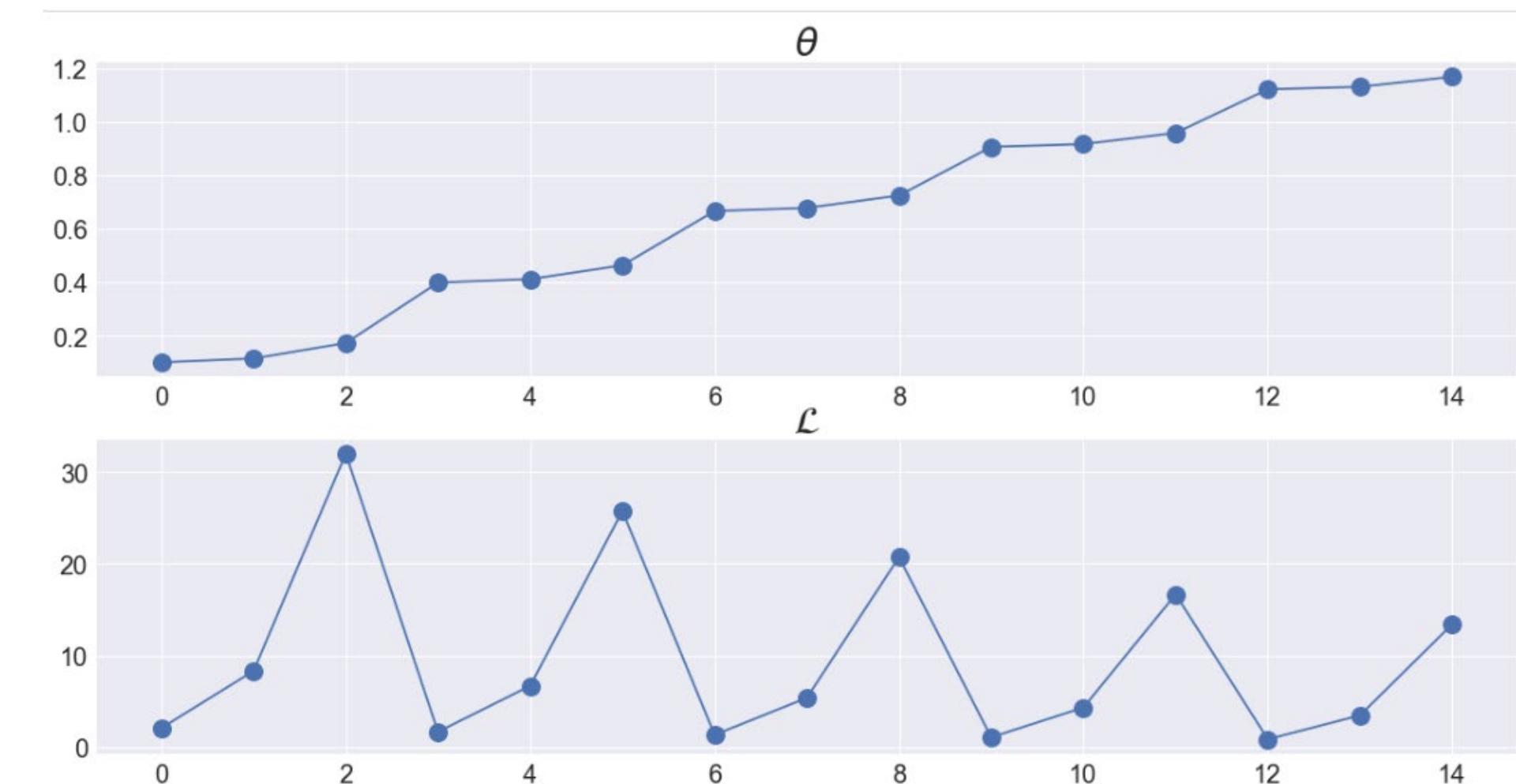
th = 0.1
lr = 0.01
epochs = 5

th_list = []
loss_list = []

for epoch in range(epochs):
    for x,y in zip(x_data, y_data):
        pred = th*x
        loss = np.power(y - pred, 2)

        th_list.append(th)
        loss_list.append(loss)
        th = th + 2*x*lr*(y - pred)
```

```
fig, ax = plt.subplots(2, 1, figsize = (20,10))
ax[0].plot(th_list, marker = 'o', markersize = 15)
ax[1].plot(loss_list, marker = 'o', markersize = 15)
ax[0].set_title(r'$\theta$', fontsize = 30)
ax[1].set_title(r'$\mathcal{L}$', fontsize = 30)
for ax_idx in range(2):
    ax[ax_idx].tick_params(axis = 'both', labelsize = 20)
```



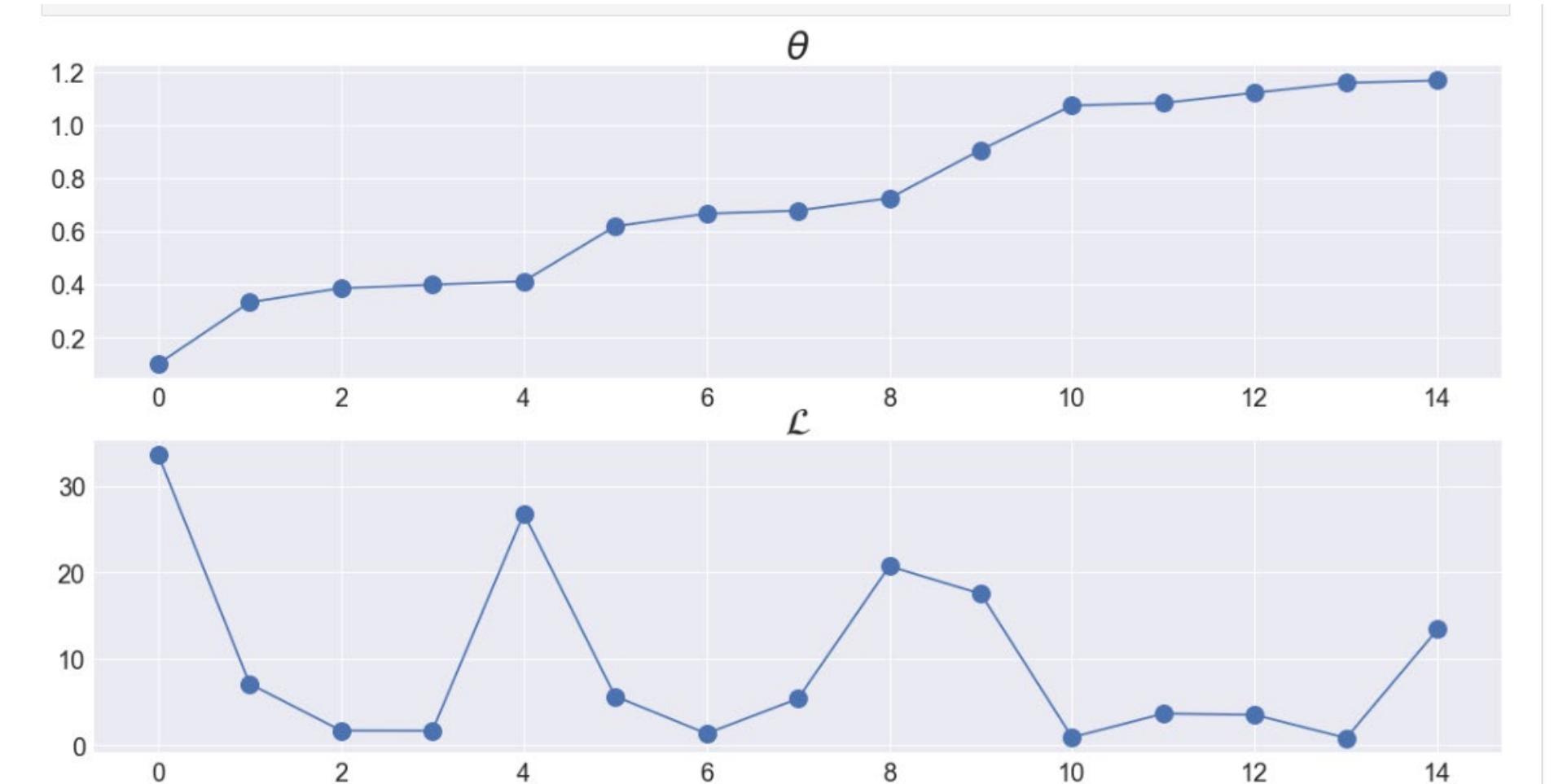
Learning with random shuffling

```
x_data = np.array([0.5, 1, 2]).reshape(-1,1)
y_data = 3*x_data
data = np.hstack((x_data, y_data))

th = 0.1
lr = 0.01
epochs = 5
th_list = []
loss_list = []
for epoch in range(epochs):
    np.random.shuffle(data)
    for x,y in data:
        pred = th*x
        loss = np.power(y - pred, 2)

        th_list.append(th)
        loss_list.append(loss)
        th = th + 2*x*lr*(y - pred)
```

```
fig, ax = plt.subplots(2, 1, figsize = (20,10))
ax[0].plot(th_list, marker = 'o', markersize = 15)
ax[1].plot(loss_list, marker = 'o', markersize = 15)
ax[0].set_title(r'$\theta$', fontsize = 30)
ax[1].set_title(r'$\mathcal{L}$', fontsize = 30)
for ax_idx in range(2):
    ax[ax_idx].tick_params(axis = 'both', labelsize = 2)
```



Epoch and batch



1 Epoch : 모든 데이터 셋을 한 번 학습

1 iteration : 1회 학습

minibatch : 데이터 셋을 batch size 크기로 조각내서 학습

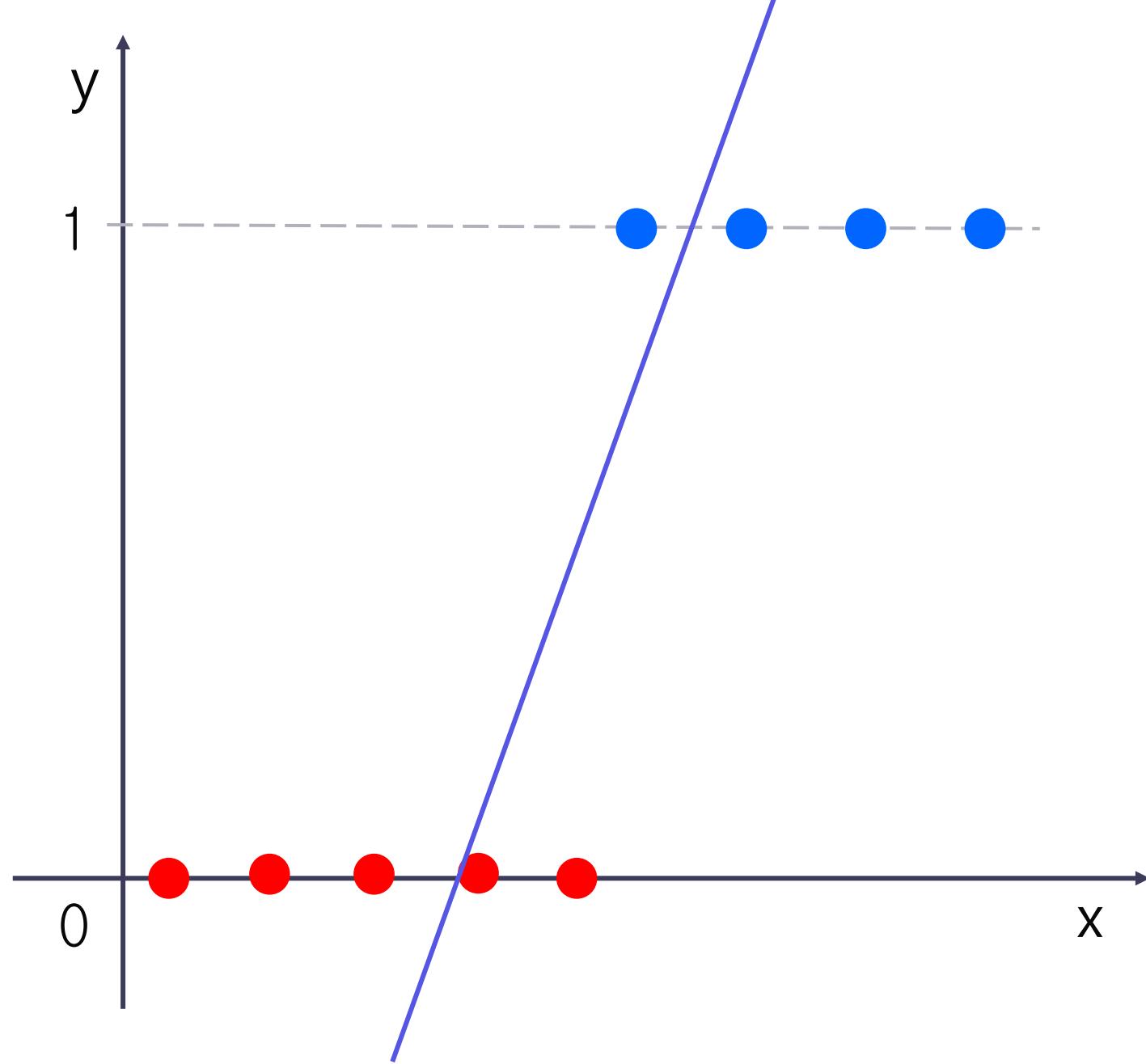
ex) 총 데이터가 100개, batch size가 10이면,

1 iteration = 10개 데이터에 대해서 학습

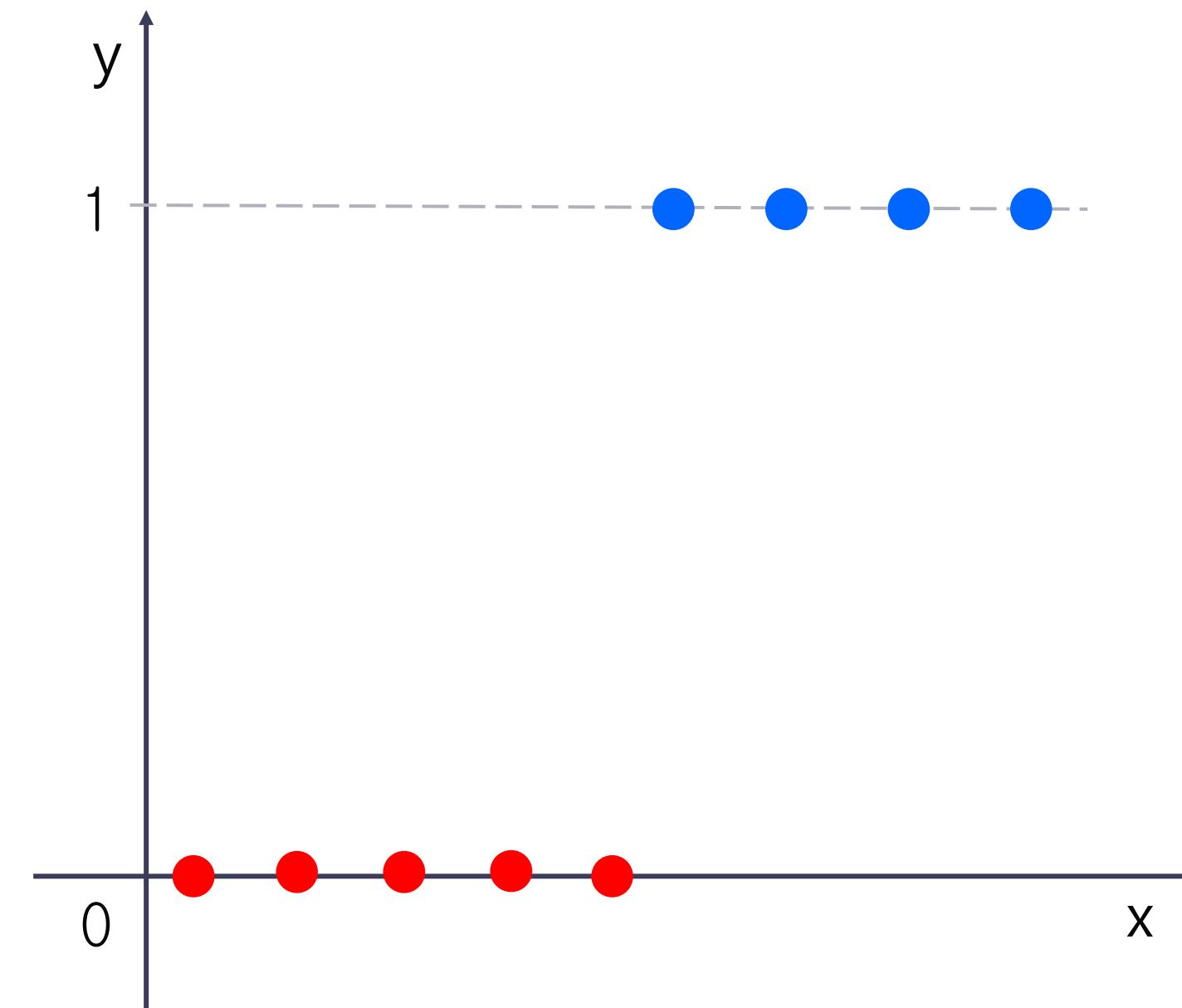
1 Epoch = $100/\text{batch size} = 10 \text{ iteration}$

로지스틱 회귀분석

사망/생존, 실패/성공, 불합격/합격



Linear regression



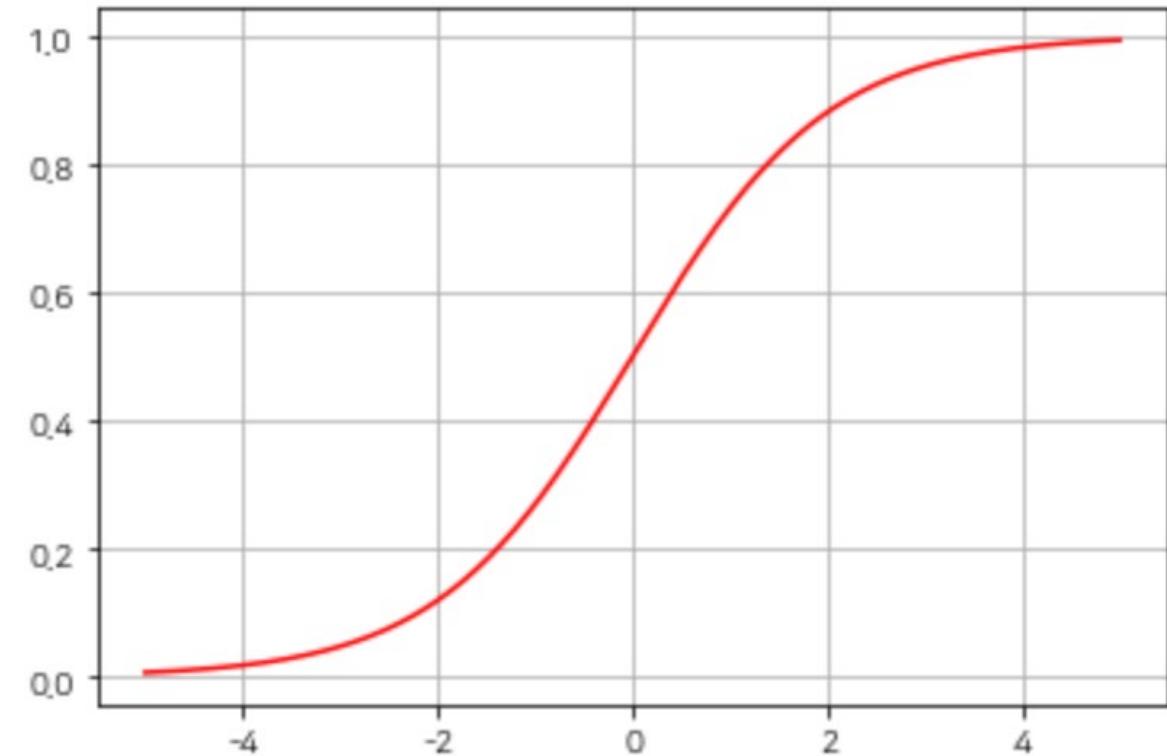
Logistic regression

승산율(오즈비, odd ratio)

odd ratio = 성공확률/실패확률

성공확률을 p 라고 하면, 오즈비 = $p/(1-p)$

로짓함수(logit function)



시그모이드 함수

• 시그모이드 함수

종속변수의 모든 실수 값에 대해

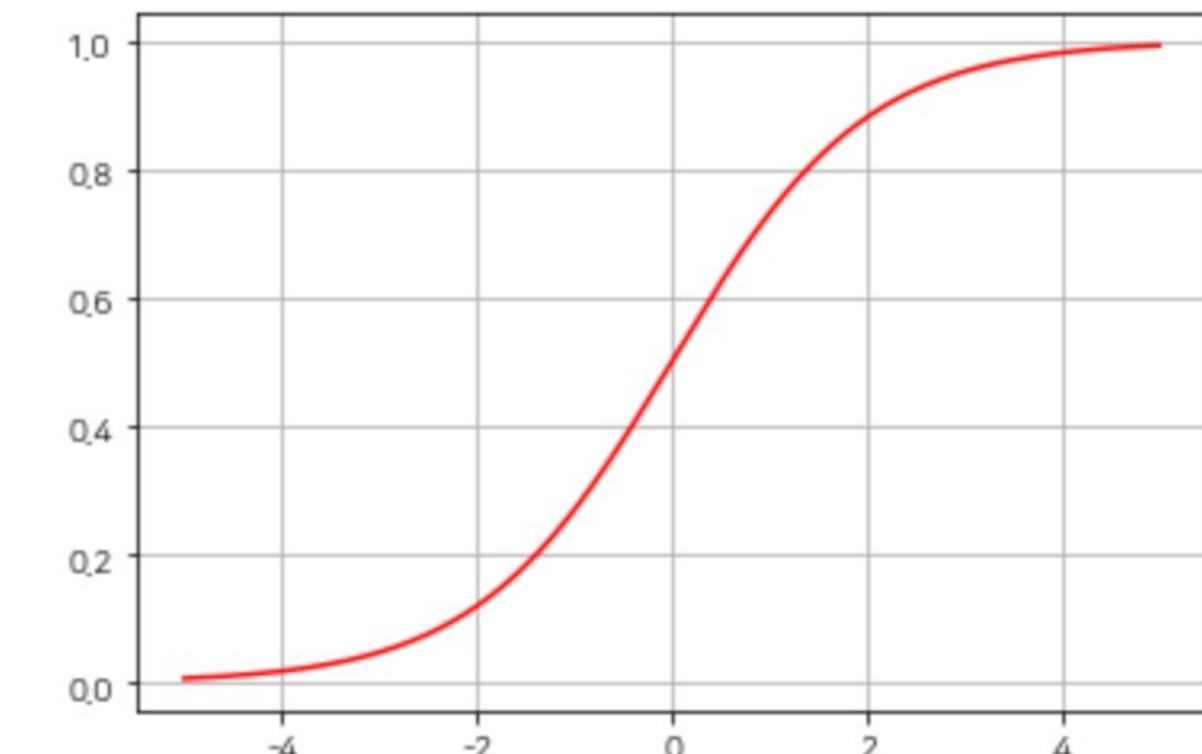
- (1) 유한한 구간 (a,b) 사이의 한정된(bounded) 값을 갖는 함수: $a < f(x) < b$
- (2) 항상 양의 기울기를 가지는 단조증가하는 함수: $a > b \rightarrow f(a) > f(b)$

을 만족하는 함수 집합을 의미한다. 예) logistic, tanh, error 함수

```
: xx = np.linspace(-5, 5, 1000)
plt.plot(xx, 1/(1+np.exp(-xx)), 'r-')
plt.grid()
```

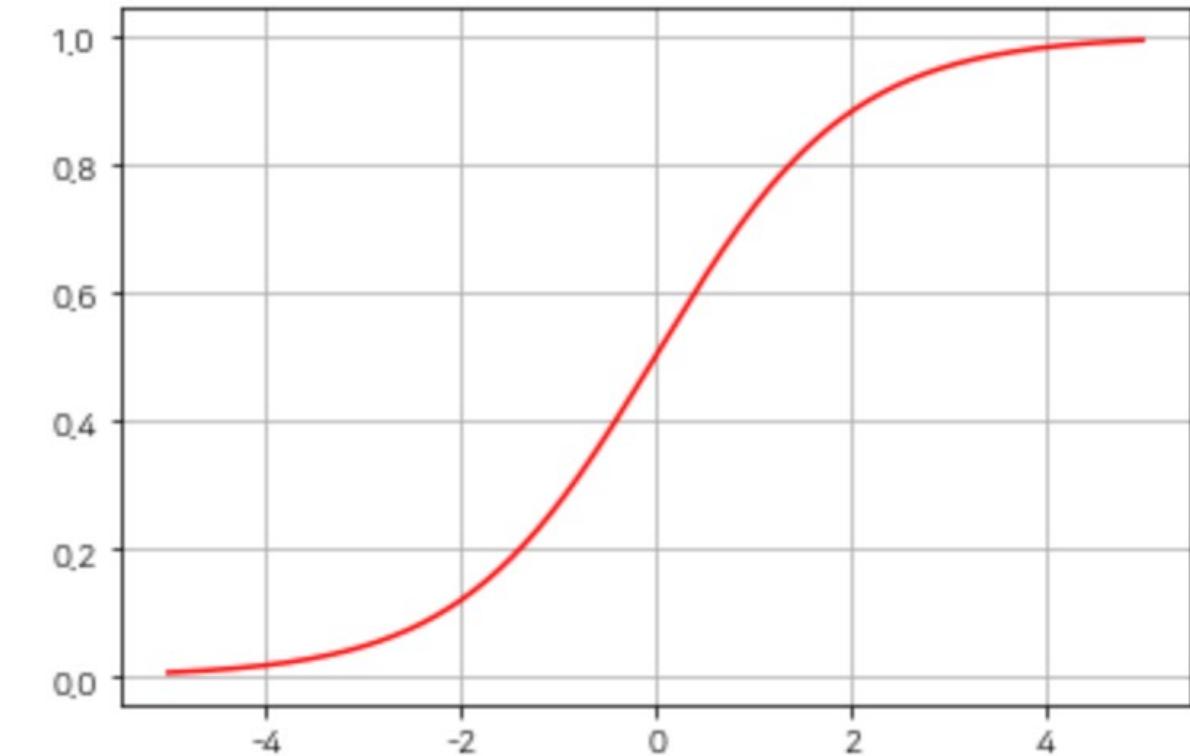
• 로지스틱 함수

$$\sigma(z) = \frac{1}{1 + exp(-z)}$$



로지스틱 함수

$-\infty$ 부터 ∞ 까지의 실수값을 0부터 1사이의 실수값으로
1 대 1 대응시키는 시그모이드함수



로지스틱 회귀분석 모형의 모수 추정



감사합니다.

